APPENDIX B

Physical Constants, Gaussian Integrals, and the Greek Alphabet

B.1 Physical Constants

We collect values of various physical constants used in the text. For dimensionful quantities involved in electricity and magnetism, we consistently use the same MKSA or SI ("Système International") units, as in Appendix A, unless specifically noted. In addition to the usual units of mass (kg), length (m), and time (s), for simplicity, we often express less familiar dimensionful quantities in terms of force (Newton, N), energy (Joule, J), and charge (Coulomb, C).

Planck's constant	$\hbar = 1.055 \times 10^{-34}$ J · s
	$= 6.582 \times 10^{-16}$ eV · s
	$h = 2\pi\hbar = 6.626 \times 10^{-34}$ J · s
speed of light (in vacuum)	$c = 2.9979 \times 10^{8}$ m/s
	$\hbar c = 1973$ eV · Å
	$= 197.3$ MeV · F
electron mass	$m_e = 9.11 \times 10^{-31}$ kg
	$m_e c^2 = 0.511$ MeV
proton mass	$m_p = 1.67 \times 10^{-27}$ kg
	$m_p c^2 = 938.3$ MeV
neutron mass	$m_n c^2 = 939.6$ MeV
muon rest mass	$m_\mu c^2 = 105.7$ MeV
fundamental charge	$e = 1.60 \times 10^{-19}$ C
electric permittivity	$\epsilon_0 = 8.85 \times 10^{-12}$ C^2/(N · m^2)
	$K = 1/4\pi\epsilon_0 = 8.98 \times 10^{9}$ N · m^2/C^2

(*Continued*)

	$Ke^2 = 2.30 \times 10^{-28}$ J · m
	$= 14.4\ eV \cdot$ Å
	$= 1.44$ MeV · F
fine structure constant	$\alpha = Ke^2\hbar c = 1/137.0$
permeability constant	$\mu_0 = 4\pi \times 10^{-7}$ N · s^2/C^2
flux quantum	$\Phi_B = 4.14 \times 10^{-15}$ T · m^2
Boltzmann constant	$k_B = 1.38 \times 10^{-23}$ J/(molecule · K)
	$= 8.617 \times 10^{-5}$ MeV/K
thermal energy at $T = 300\ K$	$k_B T = 1/39$ eV
Avogadro constant	$N_0 = 6.02 \times 10^{23}$ molecules/mole
Gas constant	$R = N_0 k_B = 8.31$ J/(mole · K)
Bohr radius	$a_0 = 0.53$ Å
Rydberg constant	$R_\infty = 1.10 \times 10^7$ m^{-1}
Rydberg energy	$E_0 = m_e c^2 \alpha^2/2 = 13.6$ eV
Gravitational constant	$G = 6.67 \times 10^{-11}$ N · m^2/kg^2
solar mass	$M_\odot = 1.99 \times 10^{30}$ kg
earth mass	$M_e = 5.98 \times 10^{24}$ kg
moon mass	$M_m = 7.36 \times 10^{22}$ kg
mean earth-sun distance	$AU = 1.50 \times 10^{11}$ m

Some useful conversion factors are:

1 Å = 10^{-10} m
1 F = 10^{-15} m
1 ly (lightyear) = 9.46×10^{15} m
1 pc (parsec) = 3.09×10^{16} m
1 eV = 1.69×10^{-19} J
1 barn = 10^{-28} m^2
1 G (Gauss) = 10^{-4} T (Tesla)

Some often used prefixes for powers of ten are:

P	peta	10^{15}	m	milli	10^{-3}
T	tera	10^{12}	μ	micro	10^{-6}
G	giga	10^{9}	n	nano	10^{-9}
M	mega	10^{6}	p	pico	10^{-12}
k	kilo	10^{3}	f	femto	10^{-15}

B.2 The Greek Alphabet

Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π
Epsilon	E	ϵ	Rho	R	ρ
Zeta	Z	ζ	Sigma	Σ	σ
Eta	H	η	Tau	T	τ
Theta	Θ	θ	Upsilon	Υ	υ
Iota	I	ι	Phi	Φ	ϕ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

B.3 Gaussian Probability Distribution

Finding the probability that a variable represented by a Gaussian probability density with mean value μ and standard deviation σ will have a value in some finite region (a, b) requires the evaluation of the "area under the curve" given by

$$\text{Prob}[x \in (a, b)] = \int_a^b dx\; P(x; \mu, \sigma) \tag{B.1}$$

where

$$P(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \tag{B.2}$$

Any such problem can be "standardized" in terms of a dimensionless variable by writing

$$z = \frac{x - \mu}{\sigma} \tag{B.3}$$

where z measures the "distance" of x away from the mean μ, in units of σ. All of the information required to evaluate such probabilities can be tabulated once and for all in the form of a cumulative probability distribution using this standardized normal random variable by calculating

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt \tag{B.4}$$

This corresponds to the probability of finding the variable t anywhere in the interval $(-\infty, z)$. It is defined such that $F(0) = 0.5$, corresponding to half the probability being on either side of μ. The integral defining $F(z)$ can be evaluated numerically and values are shown in Table B.1. They can be extended to negative values of z by using $F(-z) = 1 - F(z)$. Finally, the probability of finding the standardized variable in the interval $(z_{\min}, z_{\max})$ is given by

$$\text{Prob}[z \in (z_{\min}, z_{\max})] = F(z_{\max}) - F(z_{\min}) \tag{B.5}$$

Example B.1. Normal distributions

As an example of the use of Table B1, we can calculate the probability that a measurement of a variable corresponding to a Gaussian distribution with $\mu = 7$ and $\sigma = 2$ will find it in the interval (5.8, 9.4). The corresponding range in the standardized variables are

$$z_{\min} = \frac{5.8 - 7}{2} = -0.6 \quad \text{and} \quad z_{\max} = \frac{9.4 - 7}{2} = 1.2 \tag{B.6}$$

The probability in this interval is

$$\begin{aligned} \text{Prob}[z \in (-0.6, 1.2)] &= F(1.2) - F(-0.6) \\ &= 0.8849 - (1.0000 - 0.7257) = 0.6016 \end{aligned} \tag{B.7}$$

or about 60% of the total.

Table B.1. Values of the Cumulative Gaussian Probability Distribution Defined by the Integral in Eqn. (B.4)

z	$F(z)$	z	$F(z)$	z	$F(z)$
0.0	0.5000	1.0	0.8413	2.0	0.9722
0.1	0.5398	1.1	0.8643	2.1	0.9821
0.2	0.5793	1.2	0.8849	2.2	0.9861
0.3	0.6179	1.3	0.9032	2.3	0.9893
0.4	0.6554	1.4	0.9192	2.4	0.9918
0.5	0.6915	1.5	0.9332	2.5	0.9938
0.6	0.7257	1.6	0.9452	2.6	0.9953
0.7	0.7580	1.7	0.9554	2.7	0.9965
0.8	0.7881	1.8	0.9641	2.8	0.9974
0.9	0.8159	1.9	0.9713	2.9	0.9981
1.0	0.8413	2.0	0.9772	3.0	0.9987

B.4 **Problems**

PB.1. Verify the probabilities of measuring a Gaussian distribution in the intervals $(\mu-\sigma, \mu+\sigma)$, $(\mu-2\sigma, \mu+2\sigma)$, and $(\mu-3\sigma, \mu+3\sigma)$ as discussed in Example 4.2. How far away from μ (in terms of σ) should one go (symmetrically) on either side to have half of the probability contained under the Gaussian integral?